LITERATURE CITED

1. O. S. Esikov and E. A. Protasov, Pis'ma Zh. Tekh. Fiz., 15, No. 20, 11-14 (1989).
2. V. N. Alfeev, P. A. Bakhtin, A. A. Vasenkov, et al., in: Integrated Circuits and Microelectronic Devices Using Superconductors [in Russian], V. N. Alfeev (ed.), Moscow (1985).
3. S. I. Isaev, I. A. Kozhinov, V. I. Koftanov, et al., in: Theory of Heat and Mass Transfer [in Russian], A. I. Leont'ev (ed.), Moscow (1979).
4. I. G. Merinov, Analytic-Numerical and Experimental Studies in the Heat Physics of Nuclear Reactors [in Russian], Moscow (1983), pp. 36-43.
5. I. G. Kozhevnikov and L. A. Novitskii, Thermophysical Properties of Materials at Low Temperatures: Handbook [in Russian], Moscow (1982).
6. R. E. Krzhizhanovskii and Z. Yu. Shtern, Thermophysical Properties of Nonmetallic Materials [in Russian], Leningrad (1973).
7. V. F. Aliev, N. V. Brand, V. V. Moshchalkov, et al., Sverkhprovod. Fiz. Khim. Tekh., 2, No. 5, 29-31 (1989).
8. V. G. Veselago, A. N. Golovashkin, and O. V. Ershov, Fiz. Tverd. Tela (Leningrad), 30, No. 6, 1817-1818 (1988).

DETERMINATION OF THERMOPHYSICAL PROPERTIES OF MATERIALS WITH THE HELP OF CHARACTERISTICS OF IMAGINARY FREQUENCIES

A. V. Kozlov, A. G. Shashkov, and A. S. Trofimov

UDC 536.22.083

An effective method has been developed to determine thermophysical properties of materials from the available thermophysical measurements.

The identification of the parameters of heat transfer — the coefficients of thermal conductivity and thermal diffusivity — constitutes the essence of the internal inverse problem of thermal conduction and is reduced to determining the coefficients of the differential equation of thermal conduction from the available thermal measurements. The creation of simple engineering methods for solving such problems is a timely problem of modern thermophysics.

At present, unsteady methods of the regular regime and the initial stage of heat exchange are the best developed methods [1]. The simplicity of these methods of identification of thermophysical characteristics makes them certainly worthwhile; however, the need to perform a special experiment (to create a definite law of variation of the boundary conditions, to rapidly attain the regular regime, etc.) restricts their application. Another disadvantage is the use of temperature values obtained experimentally at fixed times in the calculated dependencies for the transfer coefficient. Possible substantial fluctuations in the measurement errors at these times can lead to substantial errors in the result. Even to a greater extent, the accuracy of determination of thermophysical characteristics depends on the quality of the experiment in algorithms [2], where the derivatives of unsteady temperatures contribute to the calculated dependencies.

In [3], an effective method of identification has been proposed, which assumes an arbitrariness in the change of the boundary conditions, and based on sufficiently simple calculated dependencies that incorporate the integrals of the temperatures taken during the experiment. Such an approach allows one to decrease the effect of measurement errors (especially, random errors) on the precision of the determination of thermophysical characteristics. However, the use by Vlasov et al. [3] of a "precise" model of heat conduction allows them to

Institute of Applied Physics, Academy of Sciences of the Belorussian SSR, Minsk. Krasnodar Polytechnic Institute. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 61, No. 1, pp. 141-145, July, 1991. Original article submitted August 20, 1990.

 0022-0841/91/6101-0922$12.50

obtain an analytical solution for a narrow class of problems (semiinfinite body, unbounded plate for boundary conditions of the first kind).

In the present study, for the identification of thermophysical characteristics, we propose to use the analysis of characteristics of imaginary frequencies of transfer functions that determine dynamic properties of heat systems [4]. The advantages of the use of characteristics of imaginary frequencies for solving direct problems of unsteady thermo-mass-conduction that involve the ease of use and high precision (relative error 1-3%) of the approximate dependencies for the temperature calculation are treated in detail in [5]. We consider the advantages of the use of characteristics of imaginary frequencies as applied to the problems of inverse coefficients on the example of the algorithm for calculating the coefficient of thermal conductivity (a = const) for an unbounded plate with boundary conditions of the first kind. Below, we compare the approximate solution obtained with the exact solution of a similar problem given in [3], and in conclusion we present the generalization of the method to other cases of heat exchange.

Mathematically the boundary-value problem is formulated in the form

$$\frac{\partial u(x,\ t)}{\partial t} = a\,\frac{\partial^2 u(x,\ t)}{\partial x^2},\ t>0,\ 0<x<R, \tag{1}$$

$$t=0,\ u=0;\ x=0,\ \partial u/\partial x=0;\ x=R,\ u=\varphi(t). \tag{2}$$

The solution of (1) and (2) in the Laplace image space is of the form

$$\bar{u}(x,\ s)=\bar{\varphi}(s)\bar{\Phi}(x,\ s),$$

where

$$\bar{\Phi}(x,\ s) = \mathrm{ch}\left(x\sqrt{\frac{s}{a}}\right)/\mathrm{ch}\left(R\sqrt{\frac{s}{a}}\right); \tag{3}$$

$$\bar{u}(x,\ s) = \int_0^\infty u(x,\ t)\exp(-st)\,dt. \tag{4}$$

The poles of the transfer function $\bar{\Phi}(x, s)$ are along the negative real semi-axis. This allows us, when solving both direct and inverse problems to consider $\bar{\Phi}(x, s)$ as a function of two real variables that vary in the intervals: $0 < x < R$, $0 < s < \infty$ (the method of characteristics of imaginary frequencies [5]). Such an approach simplifies considerably the analysis of transfer functions, providing for high accuracy of the solutions obtained.

If we measure the temperature of the left $u_e(x = 0, t)$ and the right $\varphi_e(t)$ surfaces of the plate, then by processing these curves we can construct "experimentally" the transfer function

$$\bar{\Phi}(x=0,\ \ s) = \frac{\bar{u}_e(x=0,\ s)}{\bar{\varphi}_e(s)}.$$

An analytical expression for $\bar{\Phi}(x = 0, s)$ is obtained from Eq. (3) for $x = 0$:

$$\bar{\Phi}(x=0,\ \ s) = \frac{1}{\mathrm{ch}\left(R\sqrt{\frac{s}{a}}\right)}. \tag{5}$$

By equating the right sides of the last two expressions, we obtain a calculated dependence for the coefficient of thermal conductivity:

$$a = \frac{sR^2}{\mathrm{Arch}^2\left(\frac{1}{\bar{\Phi}(x=0,\ s)}\right)}. \tag{6}$$

For more complicated situations (such as heat exchange in cylindrical, spherical, and multilayered bodies) we cannot obtain the dependence $a = f(\Phi)$ explicitly. This is what restricts the area of practical application of the method [3]. An approximation of the transfer function (5) by an approximate function $\bar{\Phi}_{ap}(s)$ allows one to expand the field of application. We consider an approximate fractionally-rational model of the first order:

$$\overline{\Phi}_{ap}(s) = \frac{1}{b_0 + b_1 \frac{R^2}{a_{ap}} s}. \tag{7}$$

Equation (7) allows one to obtain a simple expression for the coefficient of thermal conductivity that has the same structure for any geometry (the difference is only in the magnitude of the coefficients b_0 and b_1):

$$a_{ap} = \frac{b_1 s R^2}{\overline{Q}(s) - b_0}, \tag{8}$$

where $\overline{Q}(s) = \overline{\varphi}_e(s)/\overline{u}_e(x = 0, s)$.

The value of the coefficient b_0 is determined uniquely by the value of $\overline{\Phi}(x = 0, s = 0)$ (for the plate $b_0 = 1$). The value of b_1 is determined from the condition of the equality of the functions $\overline{\Phi}(x = 0, s)$ and $\overline{\Phi}_{ap}(s)$ at the point $s = s_{opt}$, which is used for the determination of a_{ap} and is characterized by the least influence of the measurement errors $\varphi_e(t)$ and $u_e(x = 0, t)$ on the accuracy of the result (8) (below, we give an algorithm for determining s_{opt}).

For such a selection of the coefficients with a decrease in the error of temperature measurements, the value of a_{ap} approaches the exact value and in the limit, in the absence of measurement errors, $a_{ap} = a$. We note that here we neglect the effect of the calculation errors in the integral characteristics $\overline{\varphi}_e(s)$ and $\overline{u}_e(x = 0, s)$ from the curves $\varphi_e(t)$ and $u_e(x = 0, t)$ on the accuracy of a_{ap}, which is justified for the case of the application of special quadrature formulas [6] that provide for high accuracy.

We determine the relative error in the calculated dependence (8) in order to select the optimal solution s_{opt}. In the first approximation

$$\frac{|a_{ap} - a|}{a_{ap}} \approx \varepsilon_a = \frac{|da_{ap}|}{a_{ap}} = \frac{1}{\overline{\varphi}_e(s)} \frac{\overline{Q}(s)\,|d\overline{\varphi}_e(s)| - \overline{Q}^2(s)|\,d\overline{u}_e(x = 0, s)|}{\overline{Q}(s) - 1}. \tag{9}$$

If the absolute measurement error for the temperature does not exceed Δ, then the differentials in Eq. (9) are estimated in the following way [3]:

$$|d\overline{\varphi}_e(s)| \leqslant \frac{\Delta}{s}; \quad |d\overline{u}_e(x = 0, s)| \leqslant \frac{\Delta}{s}$$

and the error ε_a is determined as

$$\varepsilon_a \leqslant \Delta \frac{1}{s\overline{\varphi}_e(s)} \overline{\psi}(\tilde{s}),$$

where

$$\overline{\psi}(\tilde{s}) = \frac{2 + 3b_1\tilde{s} + b_1^2\tilde{s}^2}{b_1\tilde{s}}, \quad \tilde{s} = \frac{R^2}{a_{ap}} s.$$

For two nonnegative functions the minimum value of their product is greater than or equal to the product of their least values in the same interval:

$$\min\left[\frac{1}{s\overline{\varphi}_e(s)} \overline{\psi}(\tilde{s})\right] \geqslant \min \frac{1}{s\overline{\varphi}_e(s)} \min \overline{\psi}(\tilde{s}).$$

The minimum of the first function is attained at $\overline{\varphi}(s) = \varphi_e^{max}/s$, i.e., the optimal regime (from the viewpoint of the relative error ε_a) is the regime when the temperature on the boundary $x = R$ at the time $t = 0_+$ jumps to the highest possible temperature in the experiment φ_e^{max} and then is maintained constant. The minimum of the second function is at the value of $b_1 \tilde{s}_{opt} = \sqrt{2}$, which allows us to determine $\overline{\Phi}_{ap}(\tilde{s} = \tilde{s}_{opt}) = 0.414$. We mentioned above that the value of the coefficient b_1 of the approximate model (7) is determined from the condition $\overline{\Phi}(x = 0, \tilde{s} = \tilde{s}_{opt}) = \overline{\Phi}_{ap}(\tilde{s} = \tilde{s}_{opt})$, from which it follows that $\tilde{s}_{opt} = 2.337$, $b_1 = 0.605$. An estimate of the relative error in this case is of the form

$$\varepsilon_a \leqslant 5.8 \frac{\Delta}{\varphi_e^{max}}, \tag{10}$$

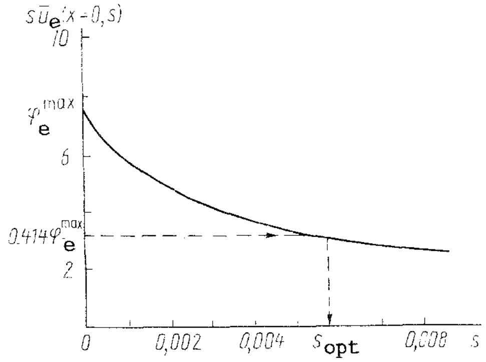

Fig. 1. A graphical determination of the value of the parameter S_{opt}.

i.e., the error in determining the coefficient of thermal conductivity in (8) is proportional to the relative temperature measurement error Δ/φ_e^{max} (an analysis of the "exact" model [3] gives a similar dependence for ε_a, however, with the proportionality constant equal to 4.8).

We demonstrate one of the algorithms for determining an optimal value of the dimensional parameter s_{opt}, which is used below for calculating a_{ap}, on the example of the analysis of experiments [7] for the plate 2R = 16.4 mm in thickness, heated uniformly to the temperature 19.5°C. Initially both surfaces of the plate were cooled down instantly to 12°C and the temperature change in the center of the plate was recorded with the help of a thermocouple. According to our estimates, the error in the temperature variation and in the plot reading [7] was not less than 5-7%.

Above we have obtained

$$s_{opt}\bar{u}_e(x=0,\ s_{opt})/\varphi_e^{max}=0.414. \tag{11}$$

The problem of the determination of s_{opt} is reduced to the graphical solution of Eq. (11). In order to do this, several values of s (usually, three are sufficient) are specified and the corresponding integral characteristics are determined:

$$s\bar{u}_e(x=0,\ s)=s\int_0^\infty u_e(x=0,\ t)\exp(-st)dt. \tag{12}$$

When calculating the characteristics, we used the quadrature formulas [6] that accounted for the behavior of the integrand (12) for large values of the exponent (st ≥ 5) and ensured high accuracy of calculations (the error less than 0.5%).

Next dots are marked on the plot (see Fig. 1) and are connected by a smooth curve. On the y axis the value of $0.414\varphi_e^{max}$ is plotted and a horizontal line is drawn to the point of intersection with the plot $s\bar{u}_e(x = 0, s)$. The projection of the intersection point on the x axis gives the desired value of s_{opt} (in our case s_{opt} = 0.0058 1/sec). By substituting s_{opt} in Eq. (8), we determine the coefficient of thermal conductivity, the value of which depends on the measurement errors least of all. In the analyzed experiment $a_{ap} = 1.54\cdot10^{-7}$ m^2/sec. For comparison, we note that calculations according to the "exact" model [3] give the value of $a = 1.63\cdot10^{-7}$ m^2/sec. Thus, the relative error in the calculation of the coefficient of thermal conductivity according to the approximate model constitutes 5.5%. This result is quite acceptable because the error due to errors in temperature measurements, from (10), is considerably larger.

In [7], the application of the method of a regular regime gave the value $a_r = 1.06\cdot10^{-7}$ m^2/sec. A considerable error of this result ($(a-a_r)/a\cdot100$ = 35%) is, in our opinion, due to strong influence of the errors in temperature measurements (they are high in experiments [7]) on the accuracy in determining a from the equations for a regular regime.

Simplicity and good accuracy of the dependencies (8) even for the comparably "rough" model and experiment allow us to recommend the method for the identification of thermophysical characteristics of materials, and also for a nondestructive complex determination of the product properties (a, λ, c, ρ).

The method described can be applied effectively also to the identification of geometric characteristics of the product parts.

We note another advantage of the method. It is that thermophysical characteristics are determined from the values of the integrals of the measured temperatures (or thermal flows), i.e., the entire range of the temperature variation in the experiment is considered. As a result we obtain the "average" value of the determined parameters in the given temperature interval. Thus, the nonlinearity of the real problem, the dependence of the thermophysical parameters on temperature, is accounted for indirectly.

In conclusion we list the main stages in solving the inverse problem of thermal conduction for the coefficients (both linear and two- and three-dimensional). All the measurements of the temperature and thermal flows that can be conducted without violation of the thermal (or technological) regime are scheduled for the considered part. The direct problem is solved (exactly or approximately) in the image space. Possible functional relationships are written for the parameters to be determined and the values to be measured. Values are determined for the parameters of the calculated dependencies for which the measurement errors affect the result least of all.

NOTATION

$u(x, t) = T(x, t) - T_0$, relative temperature, deg·K; $\overline{\Phi}(x, s)$, transfer function; s, variable of the Laplace transform, 1/sec; a, coefficient of thermal conductivity, m^2/sec.

LITERATURE CITED

1. A. V. Lykov (ed.), Methods of Heat Conduction and Temperature Conductivity [in Russian], Moscow (1973).
2. A. G. Temkin, Inverse Methods of Heat Conduction [in Russian], Moscow (1973).
3. V. V. Vlasov, Yu. S. Shatalov, and N. P. Fedorov, Teplofiz. Teplotekh., No. 36, 49-53 Kiev (1979).
4. A. G. Shashkov, System and Structural Analysis of the Process of Heat Exchange and Its Application [in Russian], Moscow (1983).
5. A. S. Trofimov, A. V. Kozlov, E. Yu. Kovalenko, and E. M. Satsko, Inzh.-Fiz. Zh., 49, No. 3, 513 (1985). Deposited at VINITI 04.04.85, No. 2299-85 Dep. (1985).
6. V. K. Lantsosh, Practical Methods of Applied Analysis [in Russian], Moscow (1965).
7. G. M. Volokhov, "An investigation of the boundary-value conditions and development of devices for determining thermophysical characteristics," Candidate's Dissertation in Technical Sciences, Minsk (1967).